

# Squeezing induced transition of long-time decay rate

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We investigate the nonclassicality of several kinds of nonclassical optical fields such as the pure or mixed single photon-added coherent states and the cat states in the photon-loss or the dephasing channels by exploring the entanglement potential as the measure. It is shown that the long-time decay of entanglement potentials of these states in photon loss channel is dependent of their initial quadrature squeezing properties. In the case of photon-loss, transition of long-time decay rate emerges at the boundary between the squeezing and non-squeezing initial non-gaussian states if log-negativity is adopted as the measure of entanglement potential. However, the transition behavior disappears if the concurrence is adopted as the measure of entanglement potential. For the case of the dephasing, distinct decay behaviors of the nonclassicality are also revealed.

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## I. INTRODUCTION

Recently, the relations between non-classicality of optical fields and the entanglement have been intensively investigated. It is shown that nonclassicality is a necessary condition for generating inseparable states via the beam splitter [1]. Based on them, a measure called the entanglement potential for quantifying the non-classicality of the single-mode optical field has been proposed [2]. The entanglement potential is defined as the entanglement achieved by 50:50 beam splitter characterized by the unitary operation  $\hat{U}_{BS} = e^{\frac{i\pi}{4}(a^\dagger b + ab^\dagger)}$  acting on the target optical mode  $a$  and the vacuum mode  $b$ . Suppose a quantum field is in the state  $\rho_a$ , then the two-mode entanglement in the state  $\hat{U}_{BS}(\rho_a \otimes |0_b\rangle\langle 0_b|)\hat{U}_{BS}^\dagger$  is regarded as its value of nonclassicality. Throughout this paper, log-negativity is explored as the measure of entanglement potential if without additional notification. The log-negativity of a density matrix  $\rho$  is defined by [3]

$$N(\rho) = \log_2 \|\rho^\Gamma\|, \quad (1)$$

where  $\rho^\Gamma$  is the partial transpose of  $\rho$  and  $\|\rho^\Gamma\|$  denotes the trace norm of  $\rho^\Gamma$ , which is the sum of the singular values of  $\rho^\Gamma$ . For any single mode pure states of quantum optical fields, their values of nonclassicality are exactly equivalent to their mixedness at characteristic time  $\gamma t = \ln 2$  of those quantum optical field in the vacuum environment (see Appendix A).

When the nonclassical optical fields propagate in the medium, they inevitably interact with their surrounding environment, which causes the dissipation or dephasing. Therefore, how a nonclassical optical field losses its non-classicality in the dephasing or dissipative channel is very desirable. The photon loss channel is the simplest one

of the general gaussian channels. Any quantum states in photon loss channel eventually asymptotically decay into the vacuum state, which is "classical" in the field of quantum optics.

The photon-added coherent state was introduced by Agarwal and Tara [4]. Recently, the single photon-added coherent state (SPACS) is experimentally prepared by Zavatta et al. and its nonclassical properties are detected by homodyne tomography technology [5]. Such a state represents the intermediate non-Gaussian state between quantum Fock state (with zero photon-number fluctuation but random phase) and classical coherent state (with well-defined amplitude and phase). In Refs.[6–8], we have investigated the entanglement potential and negativity of the Wigner function of SPACS in photon loss channel. In short time, both of them exhibit similar behaviors. However, the Wigner functions become positive when the decay time exceeds a threshold value. The threshold decay time is the same for arbitrary pure or mixed nonclassical optical fields with zero population in vacuum state [8]. Thus one can not utilize the negativity of the Wigner function to quantify the long time behavior of the non-classicality.

One of the characteristics of the photon-loss channel is any nonclassical states can not encounter finite-time sudden death of nonclassicality in photon loss channel. In this paper, we investigate the nonclassicality of several kinds of nonclassical optical fields such as the pure or mixed single photon-added coherent states and the superposition of coherent states in photon-loss or dephasing channels by exploring the entanglement potential as the measure. Different long-time decay rates of entanglement potential are found. It is shown that the photon-loss channel can cause three different kinds of long-time exponential decay rates:  $e^{-\gamma t}$  for initial quadrature squeezed states;  $e^{-2\gamma t}$  for initial nonclassical states without quadrature squeezed; and  $e^{-3\gamma t/2}$  for the boundary states between squeezing and no squeezing

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states.

This paper is organized as follows: In Sec.II, the non-classicalities measured by entanglement potential of some kinds of nonclassical states in photon-loss channel, the simplest gaussian channel, are analyzed. A initial squeezing induced transition-like behavior of the long time decay rate of the entanglement potential is revealed. In Sec.III, The influence of the dephasing on the nonclassicality of single-photon-added coherent states is investigated and shown the initial SPACS can not completely loss its nonclassicality even in the asymptotical sense. In Sec.IV, several conclusive remarks are given. In appendix A, it is shown that the entanglement potentials of one-mode pure states are completely equivalent to their mixedness achieved at the threshold decay time  $\gamma t = \ln 2$  in the photon-loss channel.

## II. SQUEEZING INDUCED TRANSITION OF LONG-TIME DECAY RATE OF NONCLASSICALITY IN PHOTON LOSS CHANNEL

Let us first briefly recall the definition of photon-added coherent states. We are interested in the dynamical behavior of the nonclassicality of the photon-added coherent states (PACS) in the dissipative channel or dephasing channel. The photon-added coherent state was firstly proposed by Agarwal and Tara [4], and was experimentally prepared by Zavatta et al. [5]. The PACS is defined by  $|\Psi_{\alpha,m}\rangle = \frac{1}{\sqrt{N(\alpha,m)}} a^{\dagger m} |\alpha\rangle$ , where  $|\alpha\rangle$  is the coherent state with the amplitude  $\alpha$  and  $a^\dagger$  ( $a$ ) is the creation operator (annihilation operator) of the optical mode.  $N(\alpha,m) = m! L_m(-|\alpha|^2)$ , where  $L_m(x)$  is the  $m$ th-order Laguerre polynomial. When the PACS evolves in the photon loss channel, the evolution of the density matrix can be described by [9]

$$\frac{d\rho}{dt} = \frac{\gamma_1}{2} (2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a), \quad (2)$$

where  $\gamma_1$  represents the dissipative coefficient. The corresponding non-unitary time evolution density matrix can be obtained as

$$\rho(t) = \frac{1}{m! L_m(-|\alpha|^2)} \sum_{k=0}^{\infty} \frac{(1 - e^{-\gamma_1 t})^k}{k!} \hat{L}(t) a^k a^{\dagger m} |\alpha\rangle \langle \alpha| a^m a^{\dagger k} \hat{L}(t), \quad (3)$$

where  $\hat{L}(t) = e^{-\frac{1}{2}\gamma_1 t a^\dagger a}$ .

For the dissipative photon-added coherent state in Eq.(3), the total output state passing through a 50/50 beam splitter characterized by the unitary operation  $e^{\frac{\pi}{4}i(a^\dagger b + ab^\dagger)}$  with a vacuum mode b can be obtained

$$\rho_{tot} = D_a(t) D_b(t) e^{-m\gamma_1 t} e^{|\alpha|^2(e^{-\gamma_1 t} - 1)} \sum_{k=0}^{\infty} \frac{(e^{\gamma_1 t} - 1)^k}{k!} \hat{E}^k \hat{E}^{\dagger m} |00\rangle \langle 00| \hat{E}^m \hat{E}^{\dagger k} D_a^\dagger(t) D_b^\dagger(t), \quad (4)$$

where  $D_a(t) = e^{\frac{\sqrt{2}}{2}\alpha(t)a^\dagger - \frac{\sqrt{2}}{2}\alpha^*(t)a}$  and  $D_b(t) = e^{\frac{\sqrt{2}i}{2}\alpha(t)b^\dagger + \frac{\sqrt{2}i}{2}\alpha^*(t)b}$  are the displacement operators of the modes a and b, respectively, where  $\alpha(t) = \alpha e^{-\gamma_1 t/2}$ .  $\hat{E} = \frac{\sqrt{2}}{2}a - \frac{\sqrt{2}i}{2}b + \alpha e^{-\frac{1}{2}\gamma_1 t}$ . The local unitary operators can not change entanglement, therefore, we only need to consider the entanglement of the mixed state given as follows:

$$\rho'_{tot} = e^{-m\gamma_1 t} e^{|\alpha|^2(e^{-\gamma_1 t} - 1)} \sum_{k=0}^{\infty} \frac{(e^{\gamma_1 t} - 1)^k}{k!} \hat{E}^k \hat{E}^{\dagger m} |00\rangle \langle 00| \hat{E}^m \hat{E}^{\dagger k}. \quad (5)$$

It is obvious that  $\hat{E}$  and  $\hat{E}^\dagger$  satisfy the commutation relation  $[\hat{E}, \hat{E}^\dagger] = 1$ . By using this commutation relation, we can simplify  $\rho'_{tot}$ .  $\hat{E}^k \hat{E}^{\dagger m} = \hat{E}^\dagger \hat{E}^k \hat{E}^{\dagger(m-1)} + k \hat{E}^{k-1} \hat{E}^{\dagger(m-1)}$ .  $\hat{E}^k \hat{E}^\dagger |00\rangle = (\alpha(t))^k \hat{E}^\dagger |00\rangle + k(\alpha(t))^{k-1} |00\rangle$ . Here, we confine our attention in the case of single quantum excitation of the classical coherent field, i.e. the single photon-added coherent state with  $m = 1$ .

For the dissipative SPACS in Eq.(3), the non-classicality of the evolving density matrix is equivalent to the non-classicality of superposition of the single-photon Fock state and vacuum state if the non-classicality is measured by entanglement potential. The evolving density matrix in Eq.(5) can also be written as

$$\begin{aligned} \rho'_{tot} = & \frac{e^{-\gamma_1 t}}{1 + |\alpha|^2} \left[ \frac{1}{2} |10\rangle \langle 10| + \frac{1}{2} |01\rangle \langle 01| \right. \\ & + \frac{i}{2} |01\rangle \langle 10| - \frac{i}{2} |10\rangle \langle 01| + \frac{\sqrt{2}}{2} \alpha^* e^{\frac{\gamma_1 t}{2}} |00\rangle \langle 10| \\ & - \frac{\sqrt{2}i}{2} \alpha^* e^{\frac{\gamma_1 t}{2}} |00\rangle \langle 01| + \frac{\sqrt{2}}{2} \alpha e^{\frac{\gamma_1 t}{2}} |10\rangle \langle 00| \\ & \left. + \frac{\sqrt{2}i}{2} \alpha e^{\frac{\gamma_1 t}{2}} |01\rangle \langle 00| + (e^{\gamma_1 t} - 1 + |\alpha|^2 e^{\gamma_1 t}) |00\rangle \langle 00| \right]. \end{aligned} \quad (6)$$

The log-negativity of the above density matrix can be analytically solved as

$$E_p = \log_2 \left( 1 - \frac{2\chi e^{-\gamma_1 t}}{1 + |\alpha|^2} \right), \quad (7)$$

where  $\chi$  is the unique negative root of the equation

$$8x^3 + (4 - 8e^{\gamma_1 t}(1 + |\alpha|^2))x^2 - (6 + 4e^{\gamma_1 t}(|\alpha|^2 - 1))x + 1 = 0. \quad (8)$$

Though the unique negative root of the above equation can be analytically obtained, its expression is complicated. From Eqs.(7-8), it is easy to see that the dissipative SPACS is always nonclassical for any large but finite decay time. In Fig.1, we have plotted the log-negativity as the function of  $\gamma_1 t$  and  $|\alpha|$ . It is shown that the entanglement potential decreases with  $\gamma_1 t$ , and also decreases with  $|\alpha|$  at short time. In Fig.2, we can see that the entanglement potential exhibits an exponential decay in

the most of the dissipative time. For  $|\alpha| \gg 1$ , the entanglement potential exhibits an exponential decay with the loss index

$$E_p(t) \simeq \log_2(1 + \frac{e^{-\gamma_1 t}}{|\alpha|^2 + 1}) \approx \frac{1}{\ln 2} \frac{e^{-\gamma_1 t}}{|\alpha|^2 + 1} \quad (9)$$

for any time scale. For  $|\alpha| = 1$ , in short time,

$$E_p(t) = \log_2[\frac{1}{3} + \frac{1}{6}e^{-\gamma_1 t} - \frac{1}{3}\cos(\theta + \frac{2\pi}{3})\sqrt{9e^{-2\gamma_1 t} + (4 - e^{-\gamma_1 t})^2}] \quad (10)$$

where

$$\theta = \frac{1}{3} \arctan(\frac{3\sqrt{6 + 3e^{\gamma_1 t}(12 + e^{\gamma_1 t}(16e^{\gamma_1 t} - 3))}}{-14 + e^{\gamma_1 t}(33 + 8e^{\gamma_1 t}(4e^{\gamma_1 t} - 3))}). \quad (11)$$

However, for long time scale,  $E_p(t) \simeq \log_2(1 + \frac{1}{4}e^{-\frac{3}{2}\gamma_1 t}) \simeq \frac{1}{4\ln 2}e^{-\frac{3}{2}\gamma_1 t}$ . For  $|\alpha| = 0$ , the EP of the single-photon Fock state follows a general curve

$$E_p(t) = \log_2(e^{-\gamma_1 t} + \sqrt{1 - 2e^{-\gamma_1 t} + 2e^{-2\gamma_1 t}}) \quad (12)$$

. Initially, EP decreases linearly with time

$$\gamma_1 t \ll 1 \quad E_p(t) \approx 1 - \frac{\gamma_1 t}{\ln 2}, \quad (13)$$

SPACS exhibits the quadrature squeezing when  $|\alpha| > 1$ . Here, we can obtain the corresponding relation between the quadrature squeezing and the entanglement potential of long time dissipative SPACS with  $|\alpha| > 1$  as

$$E_p(t) \simeq \frac{1 - \sigma_x^2}{2\ln 2} e^{-\gamma_1 t} \quad (14)$$

when  $\gamma_1 t \gg \max[1, \ln \frac{2(1+|\alpha|^2)}{(1-|\alpha|^2)^2}]$ , where

$$\begin{aligned} \sigma_x^2 &= \min_{\phi \in [0, 2\pi]} \frac{\langle \Psi_{\alpha,1} | \hat{x}_\phi^2 | \Psi_{\alpha,1} \rangle - (\langle \Psi_{\alpha,1} | \hat{x}_\phi | \Psi_{\alpha,1} \rangle)^2}{\langle \alpha | \hat{x}_\phi^2 | \alpha \rangle - (\langle \alpha | \hat{x}_\phi | \alpha \rangle)^2} \\ &= \frac{3 + |\alpha|^4}{(1 + |\alpha|^2)^2} \end{aligned} \quad (15)$$

is the minimal variance of quadrature operator  $\hat{x}_\phi = \frac{1}{2}(ae^{-i\phi} + a^\dagger e^{i\phi})$  of the initial pure SPACS. However, for the cases with  $|\alpha| < 1$ , the long time EP is given by

$$E_p(t) \approx \frac{1}{8\ln 2} \frac{e^{-2\gamma_1 t}}{(1 + |\alpha|^2)(1 - |\alpha|^2)^2}, \quad (16)$$

when  $\gamma_1 t \gg \max[1, \ln \frac{2(1+|\alpha|^2)}{(1-|\alpha|^2)^2}]$ . From the above discussions, we understand that the long-time decay behaviors of the EP of photon-loss SPACSs depend on  $|\alpha|$  and a sharp transition of the decay rate emerges at the critical value  $|\alpha| = 1$ . In Ref.[2], the EP of the general gaussian squeezing state in photon loss channel has been derived and shown that the long time behaviors of EP of general gaussian nonclassical states is proportional to  $e^{-\gamma_1 t}$ ,

but not  $e^{-2\gamma_1 t}$  or  $e^{-\frac{3}{2}\gamma_1 t}$ . Thus, we can draw the first remark that photon dissipative SPACSs with initial parameter  $|\alpha| > 1$  loss their nonclassicality like the gaussian nonclassical states in the long time scale, while photon dissipative SPACSs with initial parameter  $|\alpha| < 1$  loss their nonclassicality like the single-photon Fock state. It will be further conjectured that the long time decay rates of the nonclassicality are determined by the fact whether their states have quadrature squeezing or not.

We then consider the initial mixed states

$$\rho_M = \frac{p}{1 + |\alpha|^2} a^\dagger |\alpha\rangle \langle \alpha| a + (1 - p) |\alpha\rangle \langle \alpha| \quad (17)$$

in the photon loss channel. These initial mixed states have quadrature squeezing if  $|\alpha| > |\alpha_c| \equiv \frac{1}{\sqrt{2p-1}}$  and  $p > \frac{1}{2}$ . The non-classicality of  $\rho_M$  measured by entanglement potential can be obtained as

$$E_p(\rho_M) = \log_2(1 - \frac{2p\tau e^{-\gamma_1 t}}{1 + |\alpha|^2}), \quad (18)$$

where  $\tau$  is the unique negative root of the equation

$$8x^3 - (4 + 8\mu)x^2 - (2 - 4\mu + 16\nu)x + 1 = 0, \quad (19)$$

where

$$\begin{aligned} \mu &= \frac{1}{p} e^{\gamma_1 t} (1 + |\alpha|^2) - 1, \\ \nu &= \frac{1}{2} |\alpha|^2 e^{\gamma_1 t}. \end{aligned} \quad (20)$$

It is easy to verify that the dissipative state is always nonclassical for any large but finite time. The transition of long time behavior of EP occurs at  $|\alpha| = |\alpha_c| = \frac{1}{\sqrt{2p-1}}$  as  $p > \frac{1}{2}$ . For  $\gamma_1 t \gg 1$ ,

$$\begin{aligned} |\alpha| < |\alpha_c| : \quad E_p(t) &\propto e^{-2\gamma_1 t} \\ |\alpha| = |\alpha_c| : \quad E_p(t) &\propto e^{-\frac{3}{2}\gamma_1 t} \\ |\alpha| > |\alpha_c| : \quad E_p(t) &\propto e^{-\gamma_1 t}. \end{aligned} \quad (21)$$

Furthermore, we can also derive the

$$\sigma_x^2(p) = 1 + \frac{2p(1 + (1 - 2p)|\alpha|^2)}{(1 + |\alpha|^2)^2} \quad (22)$$

for the mixed state in Eq.(17), which is smaller than 1 when  $|\alpha| > \frac{1}{\sqrt{2p-1}}$  and  $p > \frac{1}{2}$ . In this region, the long time dynamical behavior of EP can be written as

$$\gamma_1 t \gg 1 : \quad E_p(t) \approx \frac{1 - \sigma_x^2(p)}{2\ln 2} e^{-\gamma_1 t}, \quad (23)$$

However, those states in Eq.(17) with  $|\alpha| \leq \frac{1}{\sqrt{2p-1}}$  or  $p \leq \frac{1}{2}$  are always out of the neighbor of the set of gaussian nonclassical states in photon-loss channel. In Fig.3, the  $\ln E_p$  calculated based on Eqs.(18-20) is depicted as the function of  $p$  and  $|\alpha|$  at a long decay time  $\gamma_1 t = 9$ . It explicitly outlines the transition behavior of the long time

decay rate of log-negativity for those states in Eq.(17) with or without quadrature squeezing. It is also consistent with the results in Eq.(21).

In what follows, we pay our attention to the long time decay behavior of entanglement potential of the cat states in photon loss channel, which exhibits the squeezing-dependent long time decay rate too. The cat states are described by

$$|\Psi\rangle = \frac{1}{\sqrt{N}}(|\alpha\rangle + e^{i\phi}|\alpha\rangle), \quad (24)$$

where  $N$  is the normalization constant  $N = 2 + 2e^{-2|\alpha|^2} \cos(\phi)$ .  $|\Psi\rangle$  has quadrature squeezing as  $\cos \phi > \cos \phi_c \equiv -e^{-2|\alpha|^2}$ . The dynamical behaviors of EP of the optical field initially prepared in  $|\Psi\rangle$  in the photon loss channel are calculated, and the results are shown in the Figs.(4-5). In Fig.4, we plot the EP of the photon loss cat states as the function of  $\gamma_1 t$  for several different values of  $\phi$  and  $|\alpha|$ . When  $\cos \phi > -e^{-2|\alpha|^2}$ , the initial cat states in Eq.(24) exhibit the quadrature squeezing and their long time EP can be derived as

$$\gamma_1 t \gg 1: \quad E_p(t) \approx \frac{1 - \sigma_y^2}{2 \ln 2} e^{-\gamma_1 t}, \quad (25)$$

where  $\sigma_y^2 = 1 - \frac{4|\alpha|^2(1+e^{2|\alpha|^2} \cos \phi)}{(e^{2|\alpha|^2} + \cos \phi)^2}$  is the minimal quadrature fluctuation normalized by the ordinary coherent state. When  $\cos \phi < -e^{-2|\alpha|^2}$ , the long time EP is proportional to the  $e^{-2\gamma_1 t}$ . For large  $|\alpha|$ , at intermediate times  $|2\alpha|^{-2} < \gamma_1 t \ll 1$ , the EP of the cat states are given by [2]

$$E_p(t) \approx \frac{e^{-2\gamma_1 t |\alpha|^2}}{(1 + e^{-2|\alpha|^2} \cos(\phi)) \ln 2}. \quad (26)$$

It can be observed in Fig.4 that the EPs of the cat states with the same amplitude  $|\alpha|$  but different relative phase  $\phi$  firstly decrease with the rule in Eq.(26), then at a threshold time, the bifurcation occurs and the relative phase  $\phi$  dominates the long time decay patterns of the EPs. In the inset of Fig.4, the EPs of the cat states at decay time  $\gamma_1 t = 15$  is plotted as the function of relative phase  $\phi$ . The long time decay rate of the EP slightly decreases with  $\phi$  from zero to  $\frac{\pi}{2}$  and abruptly declines at  $\phi = \arccos(-e^{-2})$ . In Fig.5, the natural logarithm of EP at time  $\gamma_1 t = 10$  is plotted as the function of  $|\alpha|$  and  $\phi$ . It is shown that the long time decay rate of the EP undergoes a transition at  $\cos \phi_c = -e^{-2|\alpha|^2}$ . The threshold line at which the long time decay rate of the EP undergoes a transition is just the boundary between quadrature-squeezing initial cat states and no-quadrature-squeezing initial cat states.

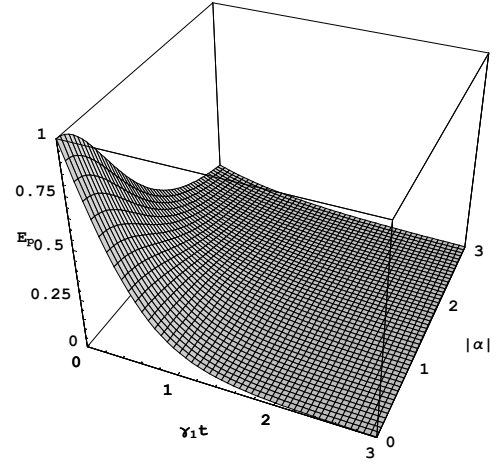


FIG. 1: The entanglement potential of the dissipative single photon-added coherent state is plotted as the function of the dissipative time  $\gamma_1 t$  and the parameter  $|\alpha|$ . Initially, EP decreases linear with time  $E_p(t) = \log_2 \left( \frac{2+|\alpha|^2}{1+|\alpha|^2} \right) - \frac{1}{\ln 2} \frac{4+|\alpha|^2}{(2+|\alpha|^2)^2} \gamma_1 t$ .

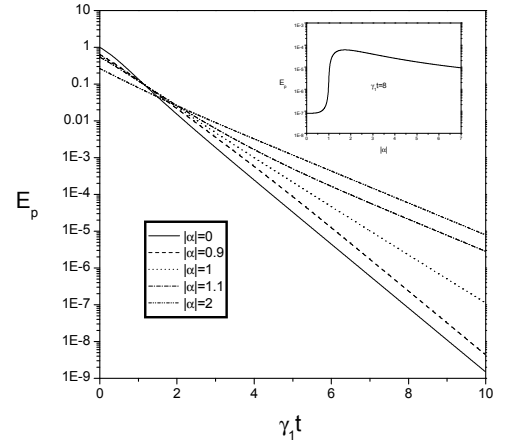


FIG. 2: The entanglement potential of the dissipative single photon-added coherent state is plotted as the function of the dissipative time  $\gamma_1 t$  for five different values of  $|\alpha|$ . It is shown that the non-classicality approximately exponentially decays with  $\gamma_1 t$  and the decay rate decreases with  $|\alpha|$ . From the inset, explicit transition-like behavior of the long time decay rate of the  $E_p$  can be found at  $|\alpha| = 1$ .

### III. NONCLASSICALITY IN THE DEPHASING CHANNEL

When the PACS evolves in the dephasing channel, the evolution of the density matrix can be described by

$$\frac{d\rho}{dt} = \frac{\gamma_2}{2} (2a^\dagger a \rho a^\dagger a - (a^\dagger a)^2 \rho - \rho (a^\dagger a)^2), \quad (27)$$

where  $\gamma_2$  represents the dephasing coefficient. The corresponding non-unitary time evolution density matrix can

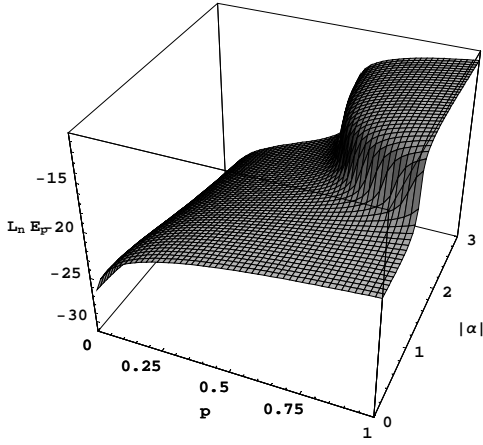


FIG. 3: The  $\ln E_p$  of the mixed states of Eq.(17) in photon loss channel is plotted as the function of the parameter  $p$  and the parameter  $|\alpha|$ .  $\gamma_1 t = 9$ .

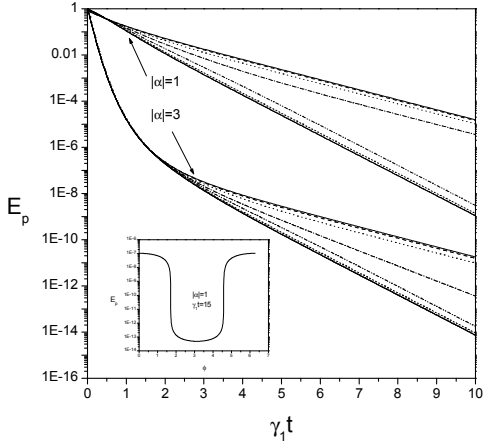


FIG. 4: The  $E_p$  of the cat states in photon loss channel are plotted as the function of  $\gamma_1 t$  with  $|\alpha| = 1$  for several different values of the relative phase  $\phi$  of the initial states. For the case with  $|\alpha| = 1$ , (Solid line)  $\phi = 0$ ; (Dash line)  $\phi = 0.5$ ; (Dot line)  $\phi = 1.0$ ; (Dash dot line)  $\phi = 1.5$ ; (Dash dot dot line)  $\phi = 2.0$ ; (Short dash line)  $\phi = 2.5$ ; (Short dot line)  $\phi = 3.0$ . For the case with  $|\alpha| = 3$ , (Solid line)  $\phi = 0$ ; (Dash line)  $\phi = 0.5$ ; (Dot line)  $\phi = 1.0$ ; (Dash dot line)  $\phi = \pi/2$ ; (Dash dot dot line)  $\phi = 2.0$ ; (Short dash line)  $\phi = 2.5$ ; (Short dot line)  $\phi = \pi$ . In the inset, the entanglement potential is shown as the function of the relative  $\phi$  with  $\gamma_1 t = 15$  and  $|\alpha| = 1$ . An abrupt transition can be found at  $\phi \approx 1.7$ .

be obtained as

$$\rho(t) = \frac{e^{-|\alpha|^2}}{m!L_m(-|\alpha|^2)} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{\alpha^k \alpha^{*n} \sqrt{(k+m)!(n+m)!}}{k!n!} e^{-\frac{1}{2}\gamma_2 t(k-n)^2} |k+m\rangle\langle n+m|. \quad (28)$$

For the dephasing photon-added coherent state in Eq.(28), the total output state passing through a 50/50 beam splitter characterized by the unitary operation

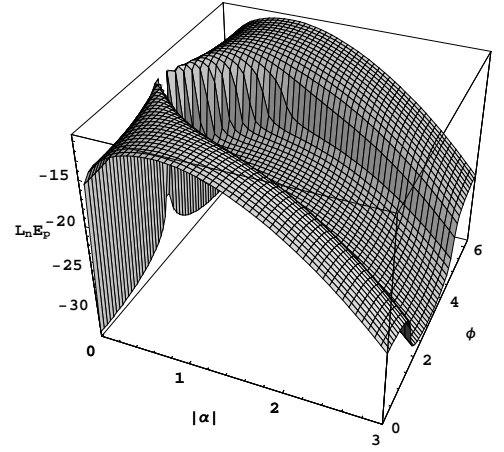


FIG. 5: The  $\ln(E_p)$  of the cat states in photon loss channel is plotted as the function of  $|\alpha|$  and  $\phi$  with  $\gamma_1 t = 10$ . An abrupt decline of the natural logarithm of entanglement potential at such a long time can be observed at  $\cos \phi = -e^{-2|\alpha|^2}$ .

$e^{\frac{\pi}{4}i(a^\dagger b + ab^\dagger)}$  with a vacuum mode  $b$  can be obtained

$$\rho_{tot} = \frac{e^{-|\alpha|^2}}{m!L_m(-|\alpha|^2)} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \sum_{j=0}^{k+m} \sum_{l=0}^{n+m} \frac{(k+m)!(n+m)! \alpha^k \alpha^{*n} i^{j-l} e^{-\frac{\gamma_2 t}{2}(k-n)^2}}{k!n! \sqrt{2^{k+n+2m}} (k+m-j)!(n+m-l)! j!l!} |k+m-j, j\rangle\langle n+m-l, l| \quad (29)$$

Here, we confine our attention in the case of single quantum excitation of the classical coherent field, i.e. the single photon-added coherent state with  $m = 1$ . For the states in Eq.(29) with  $m = 1$ , the EP is calculated and the results are displayed in Fig.6. It is shown that, in the dephasing channel, the non-classicality quantified by EP decreases with the dephasing time and stays in a stationary value depending on the seed beam intensity  $|\alpha|^2$ . The decay rate of the non-classicality at short time increases with  $|\alpha|^2$  which can be explicitly seen from Fig.7. In the inset, we also plot the total absolute decrement of EP of SPACS during the dephasing channel, which is defined by  $\Delta E_p = E_p(0) - E_p(\infty)$ . It can be found that  $\Delta E_p$  firstly increases from zero to a maximal value, then decreases with the further increase of  $|\alpha|$ . The maximal value of  $\Delta E_p$  is achieved at  $|\alpha| \approx 0.65$ . Though SPACS is considered to be very near the coherent state when  $|\alpha|$  tends to very large, the zero-time derivative of non-classicality of the dephasing SPACS exhibits an incompatible behavior with the picture that  $a^\dagger|\alpha\rangle$  tends to coherent state when  $|\alpha| \rightarrow \infty$ .

#### IV. CONCLUSIONS

In summary, we have investigated the nonclassicality of several kinds of nonclassical optical fields such as the pure or mixed single photon-added coherent states and the cat

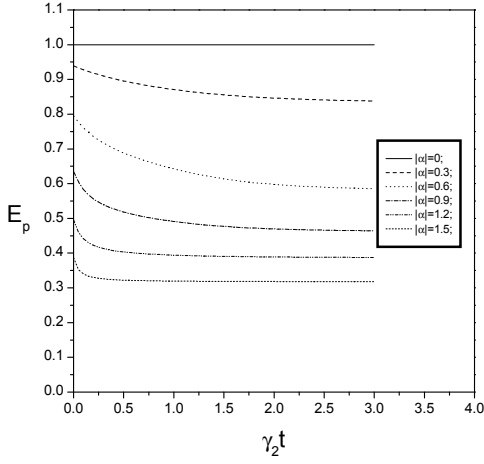


FIG. 6: The entanglement potential of the dephasing single photon-added coherent state is plotted as the function of the dissipative time  $\gamma_2 t$  for six different values of  $|\alpha|$ .

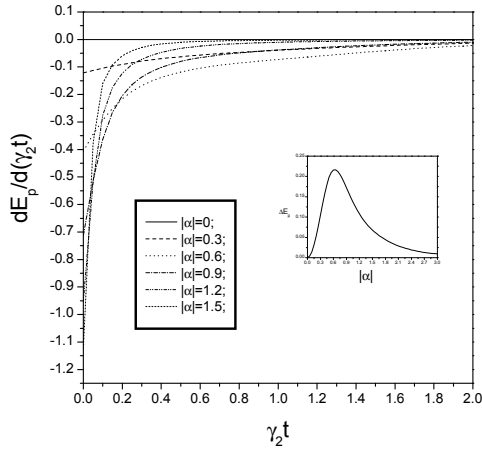


FIG. 7: The entanglement potential of the dephasing photon-added coherent state in Eq.(28) is plotted as the function of the dephasing time  $\gamma_2 t$  for six different values of  $|\alpha|$ . The absolute decrements of EP as the function of  $|\alpha|$  are shown in the inset for the dephasing SPACS.

states in the photon-loss or the dephasing channels by exploring the entanglement potential as the measure. It is shown that the long-time decay rates of entanglement potentials of these states in photon loss channel are dependent of their initial quadrature squeezing properties. In the case of the photon-loss, the transition of the long-time decay rate emerges at the boundary between the squeezing and non-squeezing initial non-gaussian states if log-negativity is adopted as the measure of entanglement potential. These examples explicitly show the long time decay of the log-negativity EP is proportional to  $\frac{1-\sigma^2}{2\ln 2}e^{-\gamma_1 t}$  with  $\sigma^2$  the minimal quadrature fluctuation normalized by coherent state of the initial states, if the initial states has quadrature squeezing. This result is a generalization of the results in Ref.[2] from gaussian squeezing states to nongaussian squeezing states. These examples also show

the long time decay of the log-negativity EP of the initial nonsqueezing state in the photon loss channel obeys the decay rule  $e^{-2\gamma_1 t}$ . For the boundary states between squeezing and nonsqueezing, their long time decay of the log-negativity EP obeys the decay rule  $e^{-\frac{3}{2}\gamma_1 t}$ .

However, this kind of transition-like behavior is measure-dependent. We have also adopt the concurrence as the measure of entanglement potential for the case of the SPACS in the photon loss channel, the transition behavior disappears. Recent studies have revealed very deep relations between the violation of uncertainty relation and the negative partial-transpose states [11]. It is conjectured this squeezing-induced transition behavior of long time decay rate of the log-negativity EP exists for more nonclassical nongaussian states in the photon loss channel if their minimal quadrature fluctuation is robust enough against the photon loss.

For the case of the dephasing, distinct decay behaviors of the nonclassicality are also revealed. For the dephasing channel, the non-classicality of SPACS is very robust and the optical field eventually evolves a stationary nonclassical state. The smaller the amplitude of initial SPACS, the larger the nonclassicality of the corresponding stationary state. The decay rate of the non-classicality at short time increases with  $|\alpha|$ .

In the Refs.[8], it has been demonstrated that quantum excitation of arbitrary optical fields can always exhibit partial negative Wigner function which will be destroyed and eventually completely disappear at the same decay time  $\gamma t_c = \ln 2$  in the photon-loss channel (vacuum environment). Based on the discussion in Appendix A, one can easily draw the following conclusion: If measured by entanglement potential, the nonclassicality of the quantum excitation of any pure single mode optical fields is completely equivalent with the necessary volume of information (quantified by von-Neumann entropy) provided by the vacuum for completely removing the negative Wigner probability distribution.

#### APPENDIX A: THE EQUIVALENCE BETWEEN NONCLASSICALITY OF A PURE NONCLASSICAL STATE AND ITS MIXEDNESS ACHIEVED IN PHOTON-LOSS CHANNEL

In this appendix, we discuss the equivalence between two concepts for any pure single-mode optical fields, nonclassicality quantified by entropic entanglement potential and photon-loss-induced mixedness quantified by von-Neumann entropy. Assume  $EEP$  is the entropic entanglement potential of an arbitrary single-mode pure state  $|\psi_a\rangle \equiv \sum_{n=0}^{\infty} d_n |n\rangle$ . According to the definition of entropic entanglement potential in Ref.[2], the  $EEP$  of any single-mode pure states are given by the von-Neumann entropy of the reduced density operator which is described by

$$\rho_r = \text{Tr}_b[\hat{U}_{BS}|\psi_a\rangle\langle\psi_a| \otimes |0_b\rangle\langle 0_b| \hat{U}_{BS}^\dagger], \quad (30)$$

where  $\hat{U}_{BS} = e^{\frac{i\pi}{4}(a^\dagger b + ab^\dagger)}$

For any single mode pure states, the measure of non-classicality is exactly equivalent to the mixedness of the quantum field undergoing the photon loss with a fixed characteristic time in the vacuum environment. For clarifying it, we only need to recall the equivalence between the master equation describing the photon loss in the vacuum environment in the interaction picture and the linear loss of beam splitter model [10]. For self-containing of the discussion, we outline its main derivation in what follows. The analytical solution of the master equation describing the photon loss,

$$\frac{\partial \rho}{\partial t} = \frac{\gamma}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a), \quad (31)$$

can be written as

$$\rho(t) = \sum_{n=0}^{\infty} \frac{(1 - e^{-\gamma t})^n}{n!} e^{-\frac{\gamma t}{2} a^\dagger a} a^n \rho(0) a^{\dagger n} e^{-\frac{\gamma t}{2} a^\dagger a}. \quad (32)$$

While the reduced density operator of mode  $a$  in the two-mode states produced by the 50/50 beam splitter injected by the single mode quantum field in the pure state  $\rho(0) \equiv |\psi_a\rangle\langle\psi_a|$  and an auxiliary vacuum state, can be calculated as

$$\begin{aligned} & \text{Tr}_b[\hat{U}_{BS}(|\psi_a\rangle\langle\psi_a| \otimes |0_b\rangle\langle 0_b|)\hat{U}_{BS}^\dagger] \\ &= \sum_{n_b=0}^{\infty} \langle n_b | \hat{U}_{BS}(|\psi_a\rangle\langle\psi_a| \otimes |0_b\rangle\langle 0_b|)\hat{U}_{BS}^\dagger | n_b \rangle \end{aligned}$$

$$\begin{aligned} &= \sum_{n_b=0}^{\infty} \frac{1}{n_b!} \langle 0_b | b^{n_b} \hat{U}_{BS}(|\psi_a\rangle\langle\psi_a| \otimes |0_b\rangle\langle 0_b|) \hat{U}_{BS}^\dagger b^{\dagger n_b} | 0_b \rangle \\ &= \sum_{n_b=0}^{\infty} \frac{1}{n_b!} \langle 0_b | \hat{U}_{BS} \hat{B}^{n_b} (|\psi_a\rangle\langle\psi_a| \otimes |0_b\rangle\langle 0_b|) \hat{B}^{\dagger n_b} \hat{U}_{BS}^\dagger | 0_b \rangle \\ &= \sum_{n_b=0}^{\infty} \frac{1}{2^{n_b} n_b!} \langle 0_b | \hat{U}_{BS} a^{n_b} (|\psi_a\rangle\langle\psi_a| \otimes |0_b\rangle\langle 0_b|) a^{\dagger n_b} \hat{U}_{BS}^\dagger | 0_b \rangle \\ &= \sum_{n_b=0}^{\infty} \frac{1}{2^{n_b} n_b!} e^{-\frac{\ln 2}{2} a^\dagger a} a^{n_b} |\psi_a\rangle\langle\psi_a| a^{\dagger n_b} e^{-\frac{\ln 2}{2} a^\dagger a}, \quad (33) \end{aligned}$$

where  $\hat{B} = \frac{\sqrt{2}}{2}b + i\frac{\sqrt{2}}{2}a$ . In the derivation of the last step, we have used the formula  $\langle 0_b | \hat{U}_{BS} | 0_b \rangle = \langle 0_b | \hat{U}_{BS}^\dagger | 0_b \rangle = e^{-\frac{\ln 2}{2} a^\dagger a}$ .

We can see that the last term in Eq.(33) exactly equal to the  $\rho(t)$  in Eq.(32) with  $t = \frac{\ln 2}{\gamma}$ . The mixedness characterized by the Von-Neumann entropy of the decohered state  $\rho(\frac{\ln 2}{\gamma})$  exactly equals to the two-mode entanglement produced by the 50/50 beam splitter injected by the single mode quantum field in the pure state  $\rho(0) \equiv |\psi_a\rangle\langle\psi_a|$  and an auxiliary vacuum state.

This equivalence implies that, the larger the nonclassicality of a single mode quantum field, the more fragile its purity against decoherence caused by the photon loss in vacuum environment.

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